submillimeter radiation with plasmas," in *Proc. Int. Conf. Submillimeter Waves and their Applications* (Atlanta, Ga.), June 1974.
[2] F. Brown, S. R. Horman, and A. Palevsky, "Characteristics of a 30-kW-peak 496 μm methyl fluoride laser," *Opt. Commun.*, vol. 9, pp. 28–30, Sept. 1973; also, T. A. DeTemple, T. K. Plant, and P. D. Coleman, "Intense superradiant emission at 496 μm from optically pumped methyl fluoride," *Appl. Phys. Lett.*, vol. 22, pp. 644–646, June 1973.
[3] F. Brown, S. Kronheim, and E. Silver, "Tunable far infrared methyl fluoride laser using transverse optical pumping," in *Proc. Int. Conf. Submillimeter Waves and their Applications* (Atlanta, Ga.), June 1974.
[4] P. V. Avizonis and R. L. Grotbeck, "Experimental and theoretical ruby laser amplifier dynamics," *J. Appl. Phys.*, vol. 37, pp. 687–693, Feb. 1966.
[5] J. F. Figueira *et al.*, "Nanosecond pulse amplifier dynamics," *Appl. Phys. Lett.*, vol. 22, pp. 216–218, Mar. 1973; also, G. T. Schappert, "Rotational relaxation effects in short-pulse CO_2 amplifiers," *Appl. Phys. Lett.*, vol. 23, pp. 319–321, Sept. 1973.
[6] T. Y. Chang and T. J. Bridges, "Laser action at 452, 496 and 541 μm in optically-pumped CH_3F," *Opt. Commun.*, vol. 1, pp. 423–426, Apr. 1970.

Optimal Design of Optics for Submillimeter Astronomy

J. E. BECKMAN AND J. A. SHAW

***Abstract*—We have designed two- and three-element shaped reflecting telescopes which minimize diffraction losses and sidelobes for submillimeter radiometric and spectrometric application. New solutions for the near-field output of an off-axis paraboloid when illuminated with a beam of considerable power taper (20 dB) show the merit of using Gaussian illumination for which the feed telescope may be explicitly designed. Conventional long-focus Cassegrain telescopes can, by using a suitably positioned off-axis conic to give a condensed focal patch, be converted for submillimeter sky chopping.**

I. INTRODUCTION

Major constraints in designing counterparts to conventional optical instruments for submillimeter work are that instrument apertures are not very large compared to wavelengths, as is the case in optical astronomy, but that technology does not readily permit the extensive use of feed horns and waveguides as in radioastronomy. Although heterodyne techniques are being extended into the range, most observers use broad-band detectors in the form of doped Ge crystal bolometers and photoconductive devices, ideally of disk shape for axial symmetry. Lens optics have the drawback that all materials having suitable refractive indexes are also strongly selectively absorbing.

There is thus the need to optimize conventional reflecting optical telescopes and auxiliary systems in order to deploy the focused radiation in the plane of the detector so that its size is minimized, thereby improving the signal to noise ratio, and the radiation sidelobes are suppressed, improving photometric performance. We have, in recent work [1], [2], proposed effective solutions for three types of observations: straightforward photometry with a Cassegrain-like telescope, amplitude- and phase-modulated Fourier spectrometry with Michelson interferometers. In this letter we also describe three-element telescopes designed for radiometry.

II. TWO-ELEMENT SYSTEMS

In reshaping the conventional Cassegrain telescope, we have found a microwave analog most useful. Pratt [3], [4] has shown how to derive a desired focal plane distribution (FPD) corresponding to a given angular beam pattern provided by a feed horn at the focus of a two-element telescope. We cannot use this method directly, since there is in any case no feed horn. Instead we solve the inverse problem of deriving the beam pattern from a given FPD, specified as that distribution which combined maximum possible axial concentration with minimum secondary diffraction maxima (sidelobes). The general expression for the FPD, $E(t)$ in terms of axial distance t, wavenumber $k = 2\pi/\lambda$ (wavelength λ), and angular beam distribution $f(\theta)$, i.e.,

$$E(t) = j(2\pi/\lambda)f_e E_0 \int_0^{\theta_0} f(\theta) \sin\theta \cos\theta J_0(kt\sin\theta)\, d\theta \qquad (1)$$

where f_e is the focal length of the system, θ_0 the cutoff angle corresponding to the focal ratio, and J_0 refers to a zero-order Bessel function. It is fortunate that in this expression the two functions $E(t)$ and $f(\theta)$ form a Hankel transform pair, so that given $E(t)$ one can derive $f(\theta)$ either from the transform equation:

$$f(\theta) = k^2 \int_0^{\infty} E(t) J_0(kt\sin\theta)\, t\, dt \qquad (2)$$

or for computing purposes, following Ruze [5] from its series approximation:

$$f(\theta) = \frac{2}{\sin^2\theta} \Sigma E\left(\frac{\beta_m/k\sin\theta_0}{J_0^2(\beta_m)}\right) J_0\left(\beta_m \left\{\frac{\sin\theta}{\sin\theta_0}\right\}\right) \qquad (3)$$

with β_m the roots of the Bessel function and discrete ordinates t_m chosen from $\beta_m = kt_m \sin\theta_0$. After various trials it was found that the FPD most suitable for transformation was given by

$$E(t) = \exp\left[-\alpha(t/t_0)^2\right] \qquad (4)$$

i.e., a Gaussian. On transformation this yields a Gaussian angular distribution

$$f(\theta) = \exp\left[-\gamma(\theta/\theta_0)^2\right] \qquad (5)$$

but this would retransform to the original sidelobeless pattern only in the limit where $\theta_0 = \pi/2$, i.e., an $f/0$ focal ratio. For realistic θ_0 values, e.g., $\theta_0 = \pi/6$ (i.e., $f/0.9$) the retransformed FPD has sidelobes, but much suppressed compared with those of a conventional diffraction-limited aperture, or Cassegrain system of equivalent focal ratio, and the energy is more compactly distributed in the focal plane.

Having designed the beam shape, the mirror shapes are then derived from ray traces plus the power conservation theorem, somewhat after Galindo [6] and Williams [7]. This is a fairly conventional procedure, but the ability to set input power equal to zero over that part of the incoming wavefront behind the secondary reflector enables power blockage to be minimized, and the efficiency thus to be optimized. Fig. 1 shows the profile of a Cassegrain-like

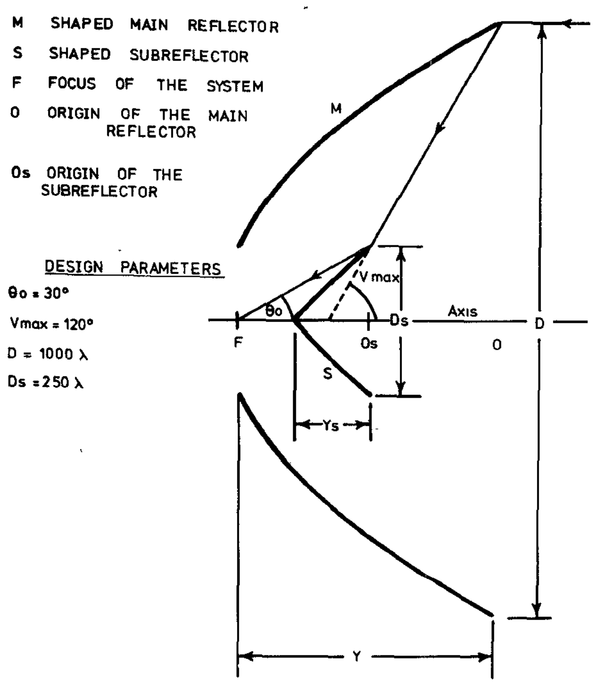

Fig. 1. Cross section of shaped two-element submillimeter reflecting telescope.

Manuscript received June 19, 1974.
J. E. Beckman was with the Department of Physics, Queen Mary College, London, England. He is now with the Astronomy Division, ESTEC, Domeinweg, Noordwijk, The Netherlands.
J. A. Shaw is with the Department of Electrical Engineering, Queen Mary College, London, England.

reflector based on these principles. Comparison with a conventional instrument shows an efficiency higher by over 10 percent and with a much more uniform response from 400- to 1000-μm wavelength, when used with a single on-axis detector of collecting diameter 1 mm.

III. THE USE OF OFF-AXIS CONICS

In order to collimate a beam from a telescope for use in an interferometer without introducing lenses, an off-axis curved reflector offers a favored solution. We have shown [2] that in the case of a paraboloid, restricting our treatment to fields along the axis, an analytical solution can be obtained using Sommerfeld's [8] version of the Kirchhoff integral for a radiating plane aperture, with a geometrical transformation. This solution enables us to compute, as a function of axial distance z from the pole of the aperture, both amplitude $E(z)$ and phase $\delta(z)$ for different types of aperture illumination. Fig. 2 illustrates this in terms of phase, showing the departure from simple geometric propagation. It is clear that when the aperture is illuminated with a 20-dB taper, the phase distribution is relatively smooth, and a corresponding result holds for amplitude. It also shows the transition from near field, where virtual geometrical propagation is maintained, to far field where the phase has shifted by $\pi/2$ compared to the geometrical path.

An overall conclusion is that by illuminating the off-axis mirror with radiation having a 20-dB Gaussian taper, the output beam is almost perfectly collimated within a distance $D^2/8\lambda$ from the aperture of diameter D. For the example of a 6-cm aperture and 1-mm wavelength this means the optical path through the spectrometer to the detector may be as long as 3.6 m. It is also necessary to shape the telescope in order to provide the required input taper, and this has been computed for a number of cases in a manner analogous to that outlined in Section II. Fig. 3 shows a polar plot of the output from a 45° off-axis paraboloid section when illuminated with 1-mm radiation at 20-dB input taper from a shaped telescope. Power concentration is along an optical axis parallel to, but not coincident with the axis of geometrical symmetry of the projected mirror. This concentration, reducing the effect of edge diffraction, limits the beam spread in the near field.

IV. THREE-ELEMENT SYSTEMS

An ability to control the output parameters of an off-axis tertiary mirror gives a considerably further scope for optimizing radiometric performance in practical cases. Fig. 4 shows, for example, that a shaped triplet telescope can be marginally more efficient and yield

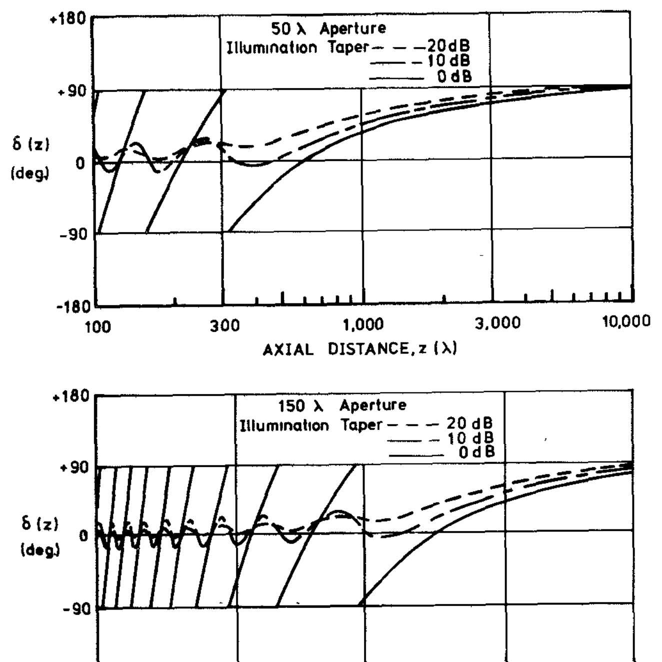

Fig. 2. Phase departures from geometrical propagation for 50λ and 150λ apertures, using wave optical calculations.

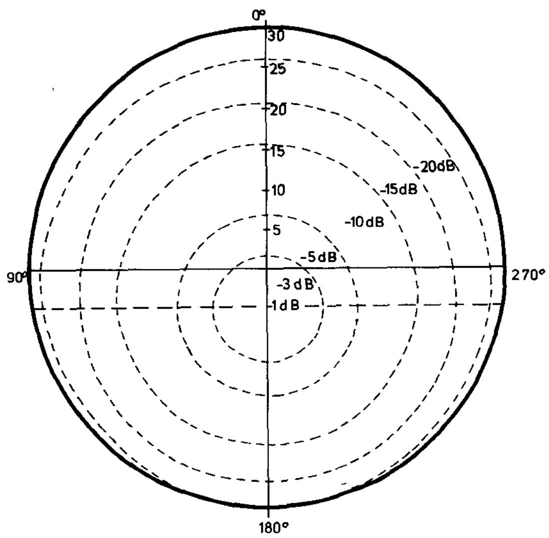

Fig. 3. Polar plot of aperture fields for 45° off-axis paraboloid of 60λ aperture, illuminated by incident field with 20-dB taper.

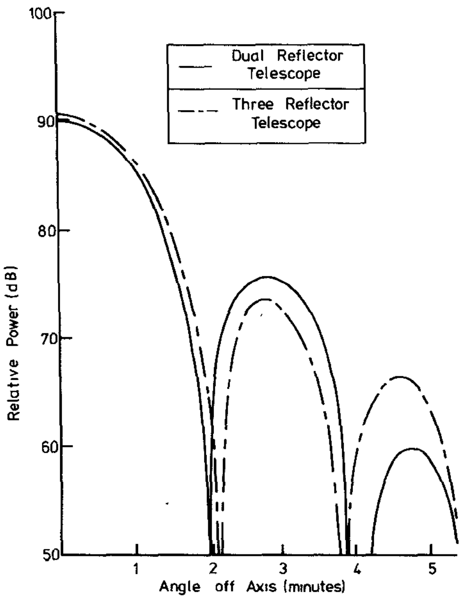

Fig. 4. Comparison of focal plane distributions for shaped doublet and triplet telescopes.

significantly improved first sidelobe suppression under diffraction-limited conditions, compared even with the shaped doublet of Section II. More generally, a fairly small off-axis conic (typical cross section < 10 cm) can be readily designed and fabricated to convert a relatively slow existing optical Cassegrain telescope, of focal ratio $f/10$ or more into an $f/3$ system with a focal patch small enough to achieve sky cancellation when used with practically available modulators. The computational techniques outlined here, and treated more fully in [1] and [2], are of wide applicability and are now being extended to off-axis fields.

REFERENCES

[1] J. E. Beckman and J. A. Shaw, "A two element telescope of high collecting efficiency for sub-millimetre astronomy," *Infrared Phys.*, vol. 12, pp. 219–234, Aug. 1972.
[2] J. A. Shaw and J. E Beckman, "Three-element optical systems for efficient interferometry at sub-millimetre wavelengths," *Infrared Phys.*, vol. 14, pp. 61–83, May 1974.
[3] T. Pratt, Marconi Internal Rep. ITM 3237, 1969.
[4] ——, "Multi-reflector antennas as communications links," presented at the Inst. Elec. Eng. Conf. 72, Earth Station Technology, 1970.
[5] J. Ruze, "Circular aperture synthesis," *IEEE Trans. Antennas Propagat.*, vol. AP-12, pp. 691–694, Nov. 1964.
[6] V. Galindo, "Design of dual reflector antennas with arbitrary phase and amplitude distributions," *IEEE Trans. Antennas Propagat.*, vol. AP-12, pp. 403–408, July 1964.
[7] W. F. Williams, "High efficiency antenna reflector," *Microwave J.*, vol. 8, pp. 79–82, July 1965.
[8] A. Sommerfeld, *Optics*. New York: Academic, 1954.

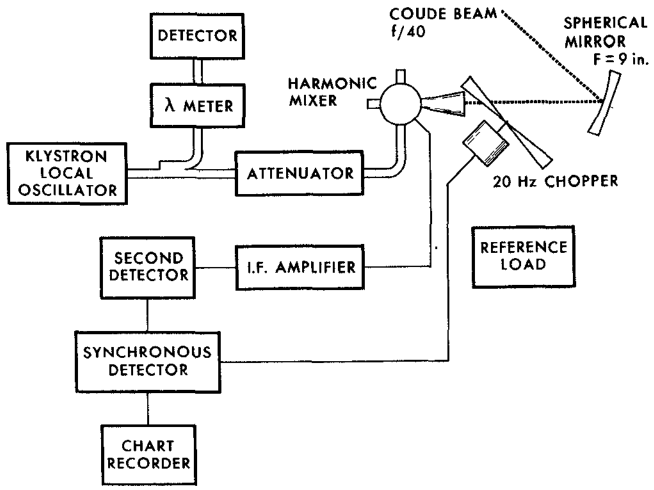

Fig. 1. Block diagram of millimeter radiometer.

Measurement of Atmospheric Attenuation at 1.3 and 0.87 mm with an Harmonic Mixing Radiometer

PAUL F. GOLDSMITH, RICHARD L. PLAMBECK, AND RAYMOND Y. CHIAO

***Abstract*—The atmospheric attenuation at 1.3 and 0.87 mm was measured above Mount Hamilton, California in the period December 5 to December 9, 1973. The measured value of the zenith attenuation varied from 1 to 5 dB at 1.3 mm over this five-day period, and was 2.5 dB at 0.87 mm on December 9, 1973. The total beamwidth of the 120″ Lick Observatory telescope used in the Coudé configuration was measured to be 3′ at 1.3 mm.**

I. EXPERIMENTAL ARRANGEMENT

The detector for this experiment was a second-harmonic mixing radiometer using a Schottky barrier diode made by G. T. Wrixon of Bell Telephone Laboratories. The local oscillator frequency was 115 GHz and the signal was accepted in two sidebands 300 MHz wide centered 1200 MHz on either side of 230 GHz. A parametric amplifier with a noise temperature of 150 K was used as the IF amplifier. The total system noise temperature was measured to be 37 000 K. An off-axis spherical mirror with a 9-in focal length was used to focus the $f/40$ Coudé beam into the $f/3$ microwave horn. A block diagram of the apparatus is shown in Fig. 1.

The system was calibrated by measuring the temperature difference between an absorber (Eccosorb AN-72) at liquid nitrogen temperature and another at ambient temperature.

II. OBSERVATIONS AND RESULTS

In order to measure the attenuation of the atmosphere and the losses in the telescope, the sky brightness temperature was measured as a function of zenith distance z. The total temperature seen by the receiver is

$$T_{\text{total}} = T_{\text{atmos}}[1 - \exp(-\tau_0 \sec z)][\exp(-\tau_{\text{tel}})] + T_{\text{tel}}[1 - \exp(-\tau_{\text{tel}})]$$

where τ_0 is the optical depth of the atmosphere at zenith and $\exp(+\tau_{\text{tel}})$ is the attenuation of the telescope. For simplicity, we assume that $T_{\text{atmos}} = T_{\text{tel}} = 280$ K and thus

$$T_{\text{total}} = 280\{1 - \exp[-(\tau_{\text{tel}} + \tau_0 \sec z)]\}.$$

Fig. 2 shows the total optical depth at 1.3 mm as a function of sec z for five zenith scans on four days. The slope of each set of data

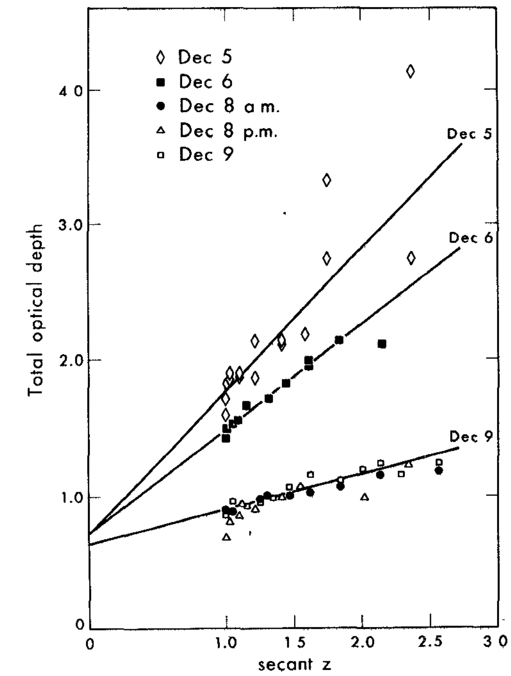

Fig. 2. Zenith scans of atmosphere at 1.3-mm wavelength.

gives τ_0, the optical depth of the atmosphere at zenith. The extrapolated intercept at sec $z = 0$ yields the effective optical depth of the telescope, $\tau_{\text{tel}} = 0.65$, implying a transmission of 52 percent. We verified this value of τ_{tel} independently by carrying the entire apparatus outdoors and measuring the sky brightness temperature at zenith without the telescope. The difference between this temperature and those measured 1 h earlier and 1 h later indoors using the telescope gives $0.35 \leq \tau_{\text{tel}} \leq 0.8$. In Table I the various τ_0 are given together with the total ground level absolute humidity at the time of the observations. It appears that ground level absolute humidity is a poor indicator of atmospheric attenuation.

The beam solid angle of the system at 1.3 mm was determined from the observed temperature of Venus, using the formula

$$\Omega_B = \Omega_{♀} \frac{T_{♀}}{T_{\text{ON}} - T_{\text{OFF}}} (1 - (T_{\text{OFF}}/280))$$

where $T_{♀} = 283$ K [1], $T_{\text{ON}} - T_{\text{OFF}}$ is the signal from Venus, measured to be 2.4 ± 0.1 K, and T_{OFF} is the sky temperature just off the source, measured to be 196 K. This yields an effective beamwidth of $3.2 \pm 0.1'$ using 32″ as the diameter of Venus.

On the last day of observations we measured the sky brightness at 0.87 mm (345 GHz) by inserting a 310-GHz high-pass filter ahead of the mixer and using the third harmonic of the local oscillator. At this frequency the system noise temperature was 280 000 K. The

Manuscript received May 8, 1974. This work was supported in part by the National Science Foundation under Grants GP 30424X and GP 38586.
The authors are with the Department of Physics, University of California, Berkeley, Calif.